# 4 Early Applied Probability: Locke, Arnauld, Leibniz, Hume

Texts

Hacking, Ian (1975*)* *The emergence of probability* Cambridge University Press, Cambridge.
Hacking, Ian (1990) *The Taming of Chance* Cambridge University Press, Cambridge.
Daston, Lorraine (1988) Classical Probability in the Enlightenment Princeton University Press, Princeton.
Locke, John(1690) An Essay Concerning Human Understanding London
Arnauld, Antoine and Pierre Nicole(1662). La logique, or l'Art de penser Clair and Girbal ed Paris 1965
Keynes, John Maynard,(1921) A treatise on probability London, MacMillan
Hume, David(1758) An enquiry concerning Human Understanding
(Downloadable from the Hume Archives on the web)

## 1 Early use of probability

Aristotle (384-322bc) Prior Analytics, Book II, Chapter 27:

*A probability and a sign are not identical, but a probability is a generally approved proposition: what men know to happen or not to happen, to be or not to be, for the most part thus and thus, is a probability, e.g. 'the envious hate', 'the beloved show affection'.*

A similar if more sophisticated use of probability appears 2000 years later in "*An Essay Concerning Human Understanding*"(1690) the empiricist philosopher John Locke (1632-1704)writes ( Book IV, chapter XV):

*Our knowledge being short, we want something else. What use to be made of this twilight state. Judgement, or assent to probability, supplies our want of knowledge.*
*Judgement is the presuming of things to be so, without perceiving it. Probability is the appearance of agreement upon fallible proofs. It is to supply our want of knowledge. Being that which makes us presume things to be true, before we know them to be.( Probability is likeliness to be true, the very notation of the word signifying such a proposition, for which there be arguments or proofs to make it pass, or be received to be true.) The grounds of probability are two: conformity to our own experience, or the testimony of others' experience. In this, all the arguments pro and con ought to be examined, before we come to a judgement..... Probability is either of sensible matter of fact, capable of human testimony, or of what is beyond the evidence of our senses. The concurrent experience of all other men with ours produces assurance approaching to knowledge ( : it froze in England last winter). Unquestionable testimony, and our own experience that a thing is for the most part so, produce confidence.( all historians write of Tiberius, that Tiberius did so). Fair testimony, and the nature of the thing indifferent, produce unavoidable assent.( a bird should fly this or that way) Experience and testimonies clashing infinitely vary the degrees of probability.(*

*proportion the assent to the different evidence and probability of the thing.: which arises and falls, according as those two foundations of credibility, viz. common observation in like cases, and particular testimonies in that particular instance, favour or contradict it. These are liable to so great a variety of contrary observations,... of the reporters, that it is impossible to reduce to precise rules the various degrees wherein men give their assent. This may only be said in general that as the arguments... shall appear in a greater or less degree to preponderate on either side; so they are fitted to produce in the mind such different entertainments, as we call belief, conjecture, guess, doubt,wavering, distrust, disbelief, etc.)*

Thus Locke, like Aristotle, contrasts probability with knowledge. He thinks of a probability as some proposition that is more probable than not, to which we might give varying degrees of assent ranging from near knowledge, through confidence,to belief, conjecture etc. But we should not expect to be any more precise than that. Locke's idea of probability comes from equating probable with provable, a proposition is probable if is 'provable', meaning there are some, but not conclusive, proofs for it. There is no hint in Locke of the continental development of numerical probability; so although the Essay was published in 1690, its probability thought predates Fermat.

## 2 Arnauld

The Port Royal *"La Logique, ou l'Art de Pense*" (1662) contains in Book IV four short chapters on probability, thought to be written by Arnauld. Probability is applied to miracles, historical events, and to future contingent events; these include games of chance for which numerical measures are given. But they also apply quantitative computations to more than mere games:

The following quote is from Hacking's treatment:

*many people who are excessively terrified when they hear thunder.... if it is only the danger of death that fills them with their extraordinary fear, it is easy to show that this is unreasonable. It would be an exaggeration to say that one in two million people is killed by a thunderstorm; there is scarcely any kind of violent death less common. Fear of harm ought to be proportional not merely to the gravity of the harm, but also the probability of the event, and since there is scarcely any kind of death more rare than death by thunderstorm, there is hardly any which ought to occasion less fear.*

Hacking comments:

*This passage shows that the author is willing to use frequency to measure probabilities of natural occurrences, and it shows he is well aware that a decision problem requires a calculation of expectation involving not only utility but also probability. 'We must reorient', he says, 'many people who conduct their lives as if they should avoid business which may have a dangerous outcome and prefer affairs which may have advantageous results. We ought to fear or hope for an event not solely in proportion to the advantage or disadvantage but also with some consideration of the likelihood of the occurrence.' There is only one case in which the probability of an event is irrelevant to deciding what to do: So long as the probabilities are not zero a strategy with infinite pay-off will always dominate all others. Since 'even the slightest chance of salvation is worth more than all the goods of the world heaped together', salvation has infinite utility. Here follows a brief statement of Pascal's argument from dominating expectation, and so ends the Port Royal Logic.*

The preceding chapter, on historical rather than future events, also applies quantitative considerations. Take the question of whether a contract witnessed by two notaries has been post-dated. Since it is certain that 999 out of 1000 duly notarized contracts have been properly dated, then 'if we know no other particulars about the contract' we ought to believe the dating is honest. 'It is incompara bly more probable that the contract before me is one of the 999 rather than the single one of the 1000 that is post-dated.' However, if we learn that the notaries are unscrupulous the document becomes less credible. If we learn that one of the parties to the

contract is thereby reported to have lent £20 000, and yet at the time of dating had only £100, then 'I should believe that there was something false about the contract.'

Note how we get a quantitative probability on the first datum but other data merely weaken this quantity in a qualitative fashion until it simply vanishes.....
This problem of mixed evidence, part counting for the hypothesis and part against, is still an open one.

## 3 Liebniz

Leibniz(1646-1715) absorbed the new probability mathematics, and made important contributions to its philosophy and applications. Leibniz's early education was as a lawyer, and his first work De Conditionibus (1665) studies conditional rights in law.

From Hacking:

***but here we notice only how it leads him to a theory of what we might now call 'partial implication '.***

*A man make a will that leaves his property for the use of B, on whose death the property is entailed to C unless B has issue. Let r be the statement that C has the right to the property, and let q state that A made such a will. Then it is not true that if q then r. Let PI be the proposition that B has living descendants, and P2 the opposite. Then q essentially subdivides into (PI-or-p2). If P2, then r, but if PI, then not-r. This is the simplest sort of case which might interest a lawyer who has to realize some property.*

***In general, consider propositions of the form 'if q then r' where q is a disjunction of mutually exclusive alternatives. Now consider a set of conditions each of which is sufficient for r. Three cases may arise. Every disjunct of q may preclude each of these conditions. In that case Leibniz calls the condition for r impossible, and we have jus nullum, no right at all. If every element of q entails some condition sufficient for r, then the condition is called necessary, and we have jus purum. However if some disjuncts entail a condition for r, while the rest entail a condition for not-r, then we have only a conditional right, and the condition is called uncertain ( incerta, in the 1665 version) or contingent (contingens, in the 1672 version). When, in q, the conditions for r are uncertain, part of q favours r and the rest favours the opposite. So we have a sort of partial implica tion: part of q implies r. When the implication is complete (that is, when the condition is necessary) Leibniz denotes it by the figure I; when the condition is impossible, he uses the cypher 0; when the condition is uncertain, the implication must be denoted by a raction. Moreover these fractions denote what are variously called 'the degrees of proof' for the right, or its 'degree of probability'.***

Thus Leibniz invented $0$ for the lowest degree of probability,
$1$ for the highest degree of probability, and also the concept of conditional probability, that all probabilities are relative to evidence.
He did not produce actual numbers for these probabilities.

# 4 Hume's Enquiry Concerning Human Understanding

David Hume(1711-1776), the Scottish empiricist, first published on chance and induction in *A Treatise of Human Nature*(1739). Revision of book I of the Treatise appeared as *An Enquiry Concerning Human Understanding*(1758), his mature thought on the subject.

The following summary of the Enquiry is extracted from www.brittannica.com.

*An Enquiry Concerning Human Understanding is an attempt to define the principles of human knowledge. It poses in logical form significant questions about the nature of reasoning in regard to matters of fact and experience, and it answers them by recourse to the principle of association. The basis of his exposition is a twofold classification of objects of awareness. In the first place, all such objects are either "impressions," data of sensation or of internal consciousness, or "ideas," derived from such data by compounding, transposing, augmenting, or diminishing. That is to say, the mind does not create any ideas but derives them from impressions. From this Hume develops a theory of meaning. A word that does not stand directly for an impression has meaning only if it brings before the mind an object that can be gathered from an impression by one of the mental processes mentioned. In the second place, there are two approaches to construing meaning, an analytical one, which concentrates on the "relations of ideas," and an empirical one, which focuses on "matters of fact." Ideas can be held before the mind simply as meanings, and their logical relations to one another can then be detected by rational inspection. The idea of a plane triangle, for example, entails the equality of its internal angles to two right angles, and the idea of motion entails the ideas of space and time, irrespective of whether there really are such things as triangles and motion. Only on this level of mere meanings, Hume asserts, is there room for demonstrative knowledge. Matters of fact, on the other hand, come before the mind merely as they are, revealing no logical relations; their properties and connections must be accepted as they are given. That primroses are yellow, that lead is heavy, and that fire burns things are facts, each shut up in itself, logically barren. Each, so far as reason is concerned, could be different: the contradictory of every matter of fact is conceivable. Therefore, any demonstrative science of fact is impossible.*

*From this basis Hume develops his doctrine about causality. The idea of causality is alleged to assert a necessary connection among matters of fact. From what impression, then, is it derived? Hume states that no causal relation among the data of the senses can be observed, for, when a person regards any events as causally connected, all that he does and can observe is that they frequently and uniformly go together. In this sort of togetherness it is a fact that the impression or idea of the one event brings with it the idea of the other. A habitual association is set up in the mind; and, as in other forms of habit, so in this one, the working of the association is felt as compulsion. This feeling, Hume concludes, is the only discoverable impressional source of the idea of causality.*

*Hume then considers the process of causal inference, and in so doing he introduces the concept of belief. When a person sees a glass fall, he not only thinks of its breaking but expects and believes that it will break; or, starting from an effect, when he sees the ground to be generally wet, he not*

*only thinks of rain but believes that there has been rain. Thus belief is a significant component in the process of causal inference. Hume then proceeds to investigate the nature of belief, claiming that he was the first to do so. He uses this term in the narrow sense of belief regarding matters of fact. He defines belief as a sort of liveliness or vividness that accompanies the perception of an idea. A belief is more than an idea; it is a vivid or lively idea. This vividness is originally possessed by some of the objects of awareness, by impressions and the simple memory images of them. By association it comes to belong to certain ideas as well. In the process of causal inference, then, an observer passes from an impression to an idea regularly associated with it. In the process the aspect of liveliness proper to the impression infects the idea, Hume asserts. And it is this aspect of liveliness that Hume defines as the essence of belief.*

*Hume does not claim to prove that the propositions, (1) that events themselves are causally related and (2) that they will be related in the future in the same ways as they were in the past, are false. He firmly believed both of these propositions and insisted that everybody else believed them, will continue to believe them, and must continue to believe them in order to survive. They are natural beliefs, inextinguishable propensities of human nature, madness apart. What Hume claims to prove is that natural beliefs are not obtained and cannot be demonstrated either by empirical observation or by reason, whether intuitive or inferential. Reflection shows that there is no evidence for them and shows also both that we are bound to believe them and that it is sensible or sane to do so. This is Hume's skepticism: it is an affirmation of that tension, a denial not of belief but of certainty.*

# 5 Hume on probability

*THOUGH there be no such thing as Chance in the world; our ignorance of the real cause of any event has the same influence on the understanding, and be gets a like species of belief or opinion.*

*There is certainly a probability, which arises from a superiority of chances on any side; and according as this superiority increases, and surpasses the opposite chances, the probability receives a proportionable increase, and be gets still a higher degree of belief or assent to that side, in which we discover the superiority. If a dye were marked with one figure or number of spots on four sides, and with another figure or number of spots on the two remaining sides, it would be more probable, that the former would turn up than the latter; though, if it had a thousand sides marked in the same manner, and only one side different, the probability would be much higher, and our belief or expectation of the event more steady and secure. This process of the thought or reasoning may seem trivial and obvious; but to those who consider it more narrowly, it may, perhaps, afford matter for curious speculation.*

*It seems evident, that, when the mind looks forward to discover the event, which may result from the throw of such a dye, it considers the turning up of each particular side as alike probable; and this is the very nature of chance, to render all the particular events, comprehended in it, entirely equal. But finding a greater number of sides concur in the one event than in the other, the mind is carried more frequently to that event, and meets it oftener, in revolving the various possibilities or chances, on which the ultimate result depends. This concurrence of several views in one particular event begets immediately, by an inexplicable contrivance of nature, the sentiment of belief, and gives that event the advantage over its antagonist, which is supported by a smaller number of views, and recurs less frequently to the mind. If we allow, that belief is nothing but a firmer and stronger conception of an object than what attends the mere fictions of the imagination, this operation may, perhaps, in some measure, be accounted for. The concurrence of these several views or glimpses imprints the idea more strongly on the imagination; gives it superior force and vigour; renders its influence on the passions and affections more sensible; and in a word, begets that reliance or security, which constitutes the nature of belief and opinion.*

*The case is the same with the probability of causes, as with that of chance. There are some causes, which are entirely uniform and constant in producing a particular effect; and no instance has ever yet been found of any failure or irregularity in their operation. Fire has always burned, and water suffocated every human creature: the production of motion by impulse and gravity is an universal law, which has hitherto admitted of no exception. But there are other causes, which have been found more irregular and uncertain; nor has rhubarb always proved a purge, or opium a soporific to every one, who has taken these medi cines. It is true, when any cause fails of producing its usual effect, philosophers ascribe not this to any irregularity in nature; but suppose, that some secret causes, in the particular structure of parts, have prevented the operation. Our reasonings, however, and conclusions concerning the event are the same as if this principle had no place. Being determined by custom to transfer the past to the future, in all our inferences; where the past has been entirely regular and uniform, we expect the event with the greatest assurance, and leave no room for any contrary supposition. But where different effects have been found to follow from*

*causes, which are to appearance exactly similar, all these various effects must occur to the mind in transferring the past to the future, and enter into our consideration, when we determine the probability of the event. Though we give the preference to that which has been found most usual, and be lieve that this effect will exist, we must not overlook the other effects, but must assign to each of them a particular weight and authority, in proportion as we have found it to be more or less frequent. It is more probable, in almost every country of Europe, that there will be frost sometime in January, than that the weather will continue open through out that whole month; though this probability varies according to the different climates, and approaches to a certainty in the more northern kingdoms. Here then it seems evident, that, when we transfer the past to the future, in order to determine the effect, which will result from any cause, we transfer all the different events, in the same proportion as they have appeared in the past, and conceive one to have existed a hundred times, for instance, another ten times, and another once. As a great number of views do here concur in one event, they fortify and confirm it to the imagination, beget that sentiment which we call belief, and give its object the preference above the contrary event, which is not supported by an equal number of experiments, and recurs not so frequently to the thought in transferring the past to the future. Let any one try to account for this operation of the mind upon any of the received systems of philosophy, and he will be sensible of the difficulty. For my part, I shall think it sufficient, if the present hints excite the curiosity of philosophers, and make them sensible how defective all com mon theories are in treating of such curious and such sublime subjects.*

## 5 Hume's skepticism about a rational basis for induction

*If we be, therefore, engaged by arguments to put trust in past experience, and make it the standard of our future judgment, these arguments must be probable only, or such as regard matter of fact and real existence according to the division above mentioned. But that there is no argument of this kind, must appear, if our explication of that species of reasoning be admitted as solid and satisfactory. We have said that all arguments concerning existence are founded on the relation of cause and effect; that our knowledge of that relation is derived entirely from experience; and that all our experimental conclusions proceed upon the supposition that the future will be conformable to the past. To endeavour, therefore, the proof of this last supposition by probable arguments, or arguments regarding existence, must be evidently going in a circle, and taking that for granted, which is the very point in question.*

*In reality, all arguments from experience are founded on the similarity which we discover among natural objects, and by which we are induced to expect effects similar to those which we have found to follow from such objects. And though none but a fool or madman will ever pretend to dispute the authority of experience, or to reject that great guide of human life, it may surely be allowed a philosopher to have so much curiosity at least as to examine the principle of human nature, which gives this mighty authority to experience, and makes us draw advantage from that similarity which nature has placed among different objects.*

*From causes which appear similar we expect similar effects. This is the sum of all our experimental conclusions. Now it seems evident that, if this conclusion were formed by reason, it would be as perfect at first, and upon one in stance, as after ever so long a course of experience. But the case is far otherwise. Nothing so like as eggs; yet no one, on account of this appearing similarity, expects the same taste and relish in all of them. It is only after a long course of uniform experiments in any kind, that we attain a firm reliance and security with regard to a particular event. Now where is that process of reasoning which, from one instance, draws a conclusion, so different from that which it infers from a hundred instances that are nowise different from that single one? This question I propose as much for the sake of information, as with an intention of raising difficulties. I cannot find, I cannot imagine any such reasoning. But I keep my mind still open to instruction, if any one will vouchsafe to bestow it on me.*

*Should it be said that, from a number of uniform experiments, we infer a connexion between the sensible qualities and the secret powers; this, I must confess, seems the same difficulty, couched in different terms. The question still recurs, on what process of argument this inference is founded? Where is the medium, the interposing ideas, which join propositions so very wide of each other? It is confessed that the colour, consistence, and other sensible qualities of bread appear not, of themselves, to have any connexion with the secret powers of nourishment and sup port. For otherwise we could infer these secret powers from the first appearance of these sensible qualities, without the aid of experience; contrary to the sentiment of all philosophers, and contrary to plain matter of fact. Here, then, is our natural state of ignorance with regard to the powers and influence of all objects. How is this remedied by experience? It only shows us a number of uniform effects, resulting from certain objects, and teaches us that those particular objects, at*

*that particular time, were endowed with such powers and forces. When a new object, endowed with similar sensible qualities, is produced, we expect similar powers and forces, and look for a like effect. From a body of like colour and consistence with bread we expect like nourishment and support. But this surely is a step or progress of the mind, which wants to be explained.*

*When a man says, I have found, in all past instances, such sensible qualities conjoined with such secret powers: And when he says, Similar sensible qualities will always be conjoined with similar secret powers, he is not guilty of a tautology, nor are these propositions in any respect the same. You say that the one proposition is an inference from the other. But you must confess that the inference is not intuitive; neither is it demonstrative: Of what nature is it, then? To say it is experimental, is begging the question.*

*For all inferences from experience suppose, as their foundation, that the future will resemble the past, and that similar powers will be conjoined with similar sensible qualities. If there be any suspicion that the course of nature may change, and that the past may be no rule for the future, all experience becomes useless, and can give rise to no inference or conclusion. It is impossible, therefore, that any arguments from experience can prove this resemblance of the past to the future; since all these arguments are founded on the supposition of that resemblance. Let the course of things be allowed hitherto ever so regular; that alone, without some new argument or inference, proves not that, for the future, it will continue so. In vain do you pretend to have learned the nature of bodies from your past experience. Their secret nature, and consequently all their effects and influence, may change, without any change in their sensible qualities. This happens some times, and with regard to some objects: Why may it not happen always, and with regard to all objects? What logic, what process or argument secures you against this supposition? My practice, you say, refutes my doubts. But you mistake the purport of my question. As an agent, I am quite satisfied in the point; but as a philosopher, who has some share of curiosity, I will not say scepticism, I want to learn the foundation of this inference. No reading, no enquiry has yet been able to remove my difficulty, or give me satisfaction in a matter of such importance. Can I do better than propose the difficulty to the public, even though, perhaps, I have small hopes of obtaining a solution? We shall at least, by this means, be sensible of our ignorance, if we do not augment our knowledge.*

*I must confess that a man is guilty of unpardonable arrogance who concludes, because an argument has escaped his own investigation, that therefore it does not really exist. I must also confess that, though all the learned, for several ages, should have employed themselves in fruit less search upon any subject, it may still, perhaps, be rash to conclude positively that the subject must, therefore, pass all human comprehension. Even though we examine all the sources of our knowledge, and conclude them un fit for such a subject, there may still remain a suspicion, that the enumeration is not complete, or the examination not accurate. But with regard to the present subject, there are some considerations which seem to remove all this accusation of arrogance or suspicion of mistake.*

*It is certain that the most ignorant and stupid peasants nay infants, nay even brute beasts-- improve by experience, and learn the qualities of natural objects, by observing the*

*effects which result from them. When a child has felt the sensation of pain from touching the flame of a candle, he will be careful not to put his hand near any candle; but will expect a similar effect from a cause which is similar in its sensible qualities and appearance. If you assert, there fore, that the understanding of the child is led into this conclusion by any process of argument or ratiocination, I may justly require you to produce that argument; nor have you any pretence to refuse so equitable a demand. You cannot say that the argument is abstruse, and may possibly escape your enquiry; since you confess that it is obvious to the capacity of a mere infant. If you hesitate, therefore, a moment, or if, after reflection, you produce any intricate or profound argument, you, in a manner, give up the question, and confess that it is not reasoning which engages us to suppose the past resembling the future, and to expect similar effects from causes which are, to appear ance, similar. This is the proposition which I intended to enforce in the present section. If I be right, I pretend not to have made any mighty discovery. And if I be wrong, I must acknowledge myself to be indeed a very backward scholar; since I cannot now discover an argument which, it seems, was perfectly familiar to me long before I was out of my cradle.*

## 6 Hume's skeptical solution

*Suppose a person, though endowed with the strongest faculties of reason and reflection, to be brought on a sudden into this world; he would, indeed, immediately observe a continual succession of objects, and one event following another; but he would not be able to discover anything farther. He would not, at first, by any reasoning, be able to reach the idea of cause and effect; since the particular powers, by which all natural operations are performed, never appear to the senses; nor is it reasonable to conclude, merely because one event, in one instance, precedes another, that therefore the one is the cause, the other the effect. Their conjunction may be arbitrary and casual. There may be no reason to infer the existence of one from the appearance of the other. And in a word, such a person, without more experience, could never employ his conjecture or reasoning concerning any matter of fact, or be assured of anything beyond what was immediately present to his memory and senses.*

*Suppose, again, that he has acquired more experience, and has lived so long in the world as to have observed familiar objects or events to be constantly conjoined together; what is the consequence of this experience? He immediately infers the existence of one object from the appearance of the other. Yet he has not, by all his experience, acquired any idea or knowledge of the secret power by which the one object produces the other; nor is it by any process of reasoning, he is engaged to draw this inference. But still he finds himself determined to draw it: and though he should be convinced that his understanding has no part in the operation, he would nevertheless continue in the same course of thinking. There is some other principle which determines him to form such a conclusion.*

*This principle is Custom or Habit. For wherever the repetition of any particular act or operation produces a propensity to renew the same act or operation, without being impelled by any reasoning or process of the under standing, we always say, that this propensity is the effect of Custom. By employing that word, we pretend not to have given the ultimate reason of such a propensity. We only point out a principle of human nature, which is universally acknowledged, and which is well known by its effects. Perhaps we can push our enquiries no farther, or pretend to give the cause of this cause; but must rest contented with it as the ultimate principle, which we can assign, of all our conclusions from experience. It is sufficient satisfaction, that we can go so far, without repining at the narrowness of our faculties because they will carry us no farther. And it is certain we here advance a very intelligible proposition at least, if not a true one, when we assert that, after the constant conjunction of two ob jects--heat and flame, for instance, weight and solidity-- we are determined by custom alone to expect the one from the appearance of the other. This hypothesis seems even the only one which explains the difficulty, why we draw, from a thousand instances, an inference which we are not able to draw from one instance, that is, in no respect, different from them. Reason is incapable of any such variation. The conclusions which it draws from consider ing one circle are the same which it would form upon surveying all the circles in the universe. But no man, having seen only one body move after being impelled by another, could infer that every other body will move after a like impulse. All inferences from experience, therefore, are effects of custom, not of reasoning.*